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Inventory is almost always a necessary evil. Hold too much of it and you have to deal with the opportunity cost of that locked capital and, sometimes, the costs associated with obsolescence. Hold too little and you might not be able to adequately service your customers. So there has to be a balance and it all starts with deciding on the “right” quantity to order every time the replenishment cycle comes around.

So where do we start? By knowing the Holy Grail, i.e., the demand for that product. But let's start with the assumption that demand is certain (no probabilistic fluctuations) and steady (no seasonality or trend). That means one can take the number of days in a given month and divide the monthly demand by the days to calculate the demand each day.

The second assumption is about the lead time--the time between order placed and order delivered—and we'll assume that the supplier lead time is zero for now. The minute you place the order with your supplier or the vendor, the stuff is shipped to you. Clearly not realistic but we'll go with that for now.

Putting in some numbers...the annual demand for a product you are trying to streamline inventory for is 60,000 units, and every order placed with the supplier is associated with a fixed cost of ordering (setup costs + shipping costs) of about \$5,000. The marginal cost of each unit is \$10 and inventory holding cost is 25% of the marginal cost of each unit each year.

Considering two scenarios...

Scenario 1: Placing an order for 30,000 units.

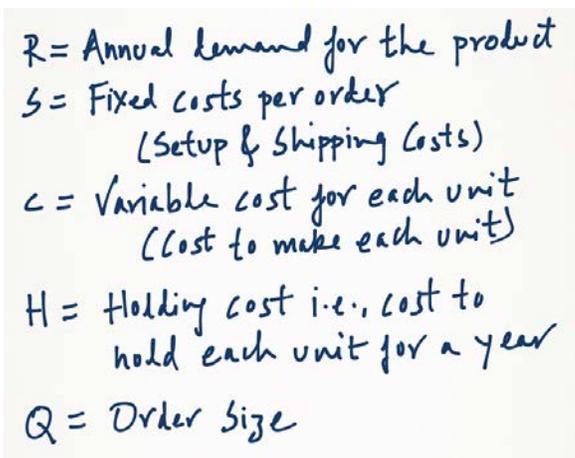
Scenario 2: Placing an order for 20,000 units.

As we move from scenario 1 to 2, the fixed costs associated with the ordering process go up. You paid 2 times annually for setup + shipping costs in scenario 1 each year and those same costs in scenario 2 are incurred 3 times. So that's not desired. But what helps in going from scenario 1 to 2 is the reduction in inventory holding costs. So fixed costs go up from scenario 1 to 2 but the variable costs (inventory holding costs) go down and hence a trade-off between the two costs is what helps decide the ideal order size.

The inventory holding cost is almost always the opportunity cost of capital used to purchase inventory and hence is expressed as a % of the unit cost that recurs every year. Money that is locked up in inventory is the money that could have been used to invest somewhere else. There is also a physical inventory holding cost associated with storage, etc., but this cost is usually a small fraction of the total holding cost and hence is oftentimes ignored.

Writing this all up...

Figure 1. Equation Variable Definitions



R = Annual demand for the product
 S = Fixed costs per order
 (Setup & Shipping Costs)
 C = Variable cost for each unit
 (Cost to make each unit)
 H = Holding cost i.e., cost to
 hold each unit for a year
 Q = Order Size

Figure 2. Annual Total Cost Equation

$$\begin{aligned}
 \text{Annual Total Cost} &= \text{Annual Ordering Cost} + \text{Annual Holding Cost} + \text{Annual Purchase Cost} \\
 \therefore ATC &= S(R/Q) + H(Q/2) + CR
 \end{aligned}$$

↑
↑
↑

Cost per order
Number of orders
Average Inventory

The objective is to minimize this total cost and hence taking a derivative of the equation above with respect to order size to calculate its effect on changes in the total cost.

The annual purchase cost is a constant because the unit cost remains unchanged regardless of the order size and hence that term goes away. There are exceptions though. If there is a quantity discount available where unit cost varies with order size then C will depend on Q and, hence, needs to be accounted for.

Figure 3. Economic Order Quantity Equation

$$\begin{aligned}
 \frac{dATC}{dQ} &= -S(R/Q^2) + H/2 + 0 \\
 S(R/Q^2) &= H/2 \\
 \therefore Q &= \sqrt{\frac{2SR}{H}} = \text{ECONOMIC ORDER QUANTITY OR EOQ}
 \end{aligned}$$

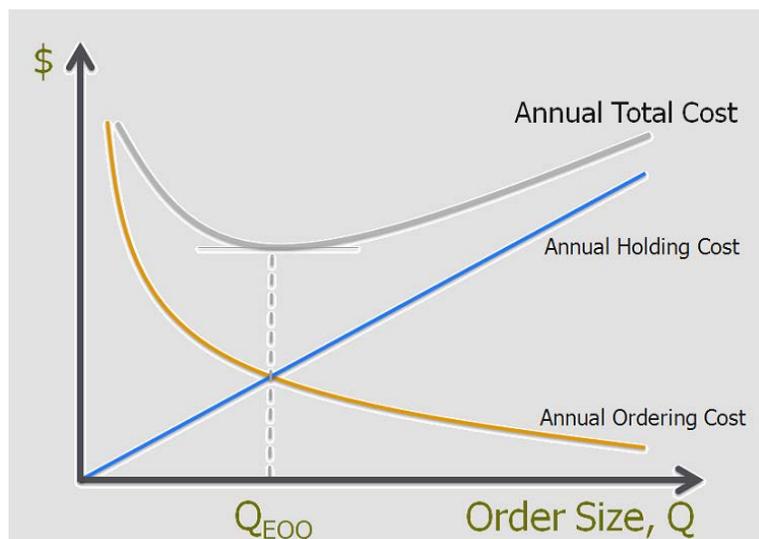
Costs, costs and only costs are being discussed here because we are making one fundamental assumption - the price of the product and hence the revenues do not change, and since they don't change, the only way profits can be maximized is by reducing the total cost.

Back to the equations above, as the order size increases, the annual ordering cost, of course, should decrease, and the negative sign in front of the first term confirms that. As the order size increases, the annual holding cost increases; hence the positive sign in front of the second term. The third term, as discussed before, is zero because an order size change has no influence on the purchase cost. The first term is the marginal benefit of increasing the order size. The second term is the marginal cost of increasing the order size. Equating that differential equation (wait, what...) with zero gives the ideal order quantity with the lowest annual total cost. That is, the optimal or the most economic order quantity (Q_{EOQ}) is when marginal benefit equals marginal cost.

If the shipping rates go up, the Q_{EOQ} goes up as well because we will be placing fewer numbers of orders. As the inventory holding costs go up, our natural response is to decrease the order size and that is corroborated by the EOQ equation. As demand goes up, of course the order size goes up. Broadly matches our intuition.

Another thing to note is the non-linearity of the relationship between EOQ and the other factors. As the fixed cost of ordering doubles, say, from S to $2S$, the Q_{EOQ} only increases by the square root of 2. So doubling the fixed costs of ordering only increases the optimal order quantity by about 40%. That is evident from the wider, flatter behavior of the total cost curve, especially around the optimal point, which, in fact, proves to be a very desirable trait about this relationship between cost and the order size.

Figure 4. Cost Curves



Future demand is usually based upon some kind of a forecast and hence is likely prone to errors. But, as it turns out, even with these errors, the optimal solution does not change much, due again to the flatness around the Q_{EOQ} point on the total cost curve. Hence, the EOQ model is deemed to be a fairly robust model even in the face of these uncertainties.

$Q_{EOQ}/2$ is the average inventory held for a given product over time and hence anything we can do to reduce the Q_{EOQ} will have a direct impact on the bottom line. And there are not that many levers to pull. Holding costs and product demand are beyond the scope of what an operations engineer can control, so the only lever left to pull is the change (or reduction) in fixed costs associated with each order. You make that happen by collaborating and negotiating with the suppliers, and you reduce the Q_{EOQ} , which, in turn, reduces the average inventory on hand at any given time. And then you are golden.



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